Encodability and Separation for a Reflective Higher-Order Calculus

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What is a name?

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"Assume a countably infinite set $\mathcal{N}$ of atomic entities x, y, z ..."

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"Assume a countably infinite set $\mathcal{N}$ of atomic entities x, y, z ..."

But is that a reasonable assumption?

The π-calculus

$P ::=$	$\mathbf{0}$	nil
$\mid$	$P_1 \mid P_2$	parallel
$\mid$	$x(y).P$	input
$\mid$	$\overline{x}{<}z{>}$	output (async)
$\mid$	$!P$	replication
$\mid$	$(\nu x)P$	restriction

Agenda

- The ρ-calculus

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- The ρ-calculus
- How not to encode the π-calculus

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- The ρ-calculus
- How not to encode the π-calculus
- How to correctly encode the π-calculus

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- The ρ-calculus
- How not to encode the π-calculus
- How to correctly encode the π-calculus
- Why the converse is impossible

The Reflective Higher-Order Calculus

(Meredith & Radestock, 2005)

From π-calculus to ρ-calculus

$P ::=$	$\mathbf{0}$	nil
$\mid$	$P \mid P$	parallel
$\mid$	$x(y).P$	input
$\mid$	$\overline{x}\langle z \rangle$	output
$\mid$	$!P$	replication
$\mid$	$(\nu x)P$	restriction

From π-calculus to ρ-calculus

$$
\begin{array}{rcll}
P & ::= & \mathbf{0} & \text{nil} \\
  & | & P \mid P & \text{parallel} \\
  & | & x(y).P & \text{input} \\
  & | & \overline{x}{<}z{>} & \text{output} \\
  & | & !P & \text{replication} \\
  & | & (\nu x)P & \text{restriction}
\end{array}
$$

From π-calculus to ρ-calculus

P	$::= \mathbf{0}$	nil
	$\mid\ P \mid P$	parallel
	$\mid\ x(y).P$	input
	$\mid\ \overline{x}{<}z{>}$	output
	$\mid\ !P$	replication
x	$::= \ulcorner P \urcorner$	quote

From π-calculus to ρ-calculus

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\begin{array}{rcll}
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  & | & !P & \text{replication} \\
x & ::= & \ulcorner P \urcorner & \text{quote}
\end{array}
$$

From π-calculus to ρ-calculus

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\begin{array}{rcll}
P & ::= & \mathbf{0} & \text{nil} \\
 & \mid & P \mid P & \text{parallel} \\
 & \mid & x(y).P & \text{input} \\
 & \mid & \overline{x}\langle z\rangle & \text{output} \\
 & \mid & \urcorner x \ulcorner & \text{drop} \\
x & ::= & \ulcorner P \urcorner & \text{quote}
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$$

From π-calculus to ρ-calculus

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 & \mid & \urcorner x \ulcorner & \text{drop} \\
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From π-calculus to ρ-calculus

$$
\begin{array}{rcll}
P & ::= & \mathbf{0} & \text{nil} \\
 & | & P \mid P & \text{parallel} \\
 & | & x(y).P & \text{input} \\
 & | & x \langle\!| P |\!\rangle & \text{lift} \\
 & | & \urcorner x \ulcorner & \text{drop} \\
x & ::= & \ulcorner P \urcorner & \text{quote}
\end{array}
$$

From π-calculus to ρ-calculus

$$
\begin{array}{rcll}
P & ::= & \mathbf{0} & \text{nil} \\
& | & P \mid P & \text{parallel} \\
& | & \ulcorner P \urcorner (\ulcorner P \urcorner).P & \text{input} \\
& | & \ulcorner P \urcorner \langle\!| P |\!\rangle & \text{lift} \\
& | & \urcorner \ulcorner P \urcorner \ulcorner & \text{drop}
\end{array}
$$

Example 1

$$x \langle\!| P |\!\rangle \mid x(y).(y \langle\!| \ulcorner y \urcorner \mid Q |\!\rangle \mid \ulcorner y \urcorner)$$

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$$
\begin{aligned}
& x \llparenthesis P \rrparenthesis \mid x(y).(y \llparenthesis \urcorner y \ulcorner \mid Q \rrparenthesis \mid \urcorner y \ulcorner) \\
\to\ & \left(y \llparenthesis \urcorner y \ulcorner \mid Q \rrparenthesis \mid \urcorner y \ulcorner \right) \{ \ulcorner P \urcorner / y \}
\end{aligned}
$$

Example 1

$$x\langle\!|P|\!\rangle \mid x(y).(y\langle\!|\ulcorner y\urcorner \mid Q|\!\rangle \mid \ulcorner y\urcorner)$$
$$\to \ulcorner P\urcorner\langle\!|P \mid Q|\!\rangle \mid P$$

Definition (Reflection)

Why *reflective*?

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When a program is *reflective* it can

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When a program is *reflective* it can

- Turn code into data

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When a program is *reflective* it can

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- compute with it

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- even modify it

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When a program is *reflective* it can

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- even modify it
- and reinstantiate it as running code

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communicate

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communicate

When a program is *reflective* it can

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- ~~compute~~ with it
- ~~even modify it~~
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Example 2: Replication

$$D(x) \triangleq x\,(y)\,.\,(\ulcorner y \urcorner \mid x\,\langle\!|\ulcorner y \urcorner|\!\rangle)$$

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$$D(x) \triangleq x\,(y).(\ulcorner y \urcorner \mid x\,\langle\!|\ulcorner y \urcorner|\!\rangle)$$

$$\begin{aligned} & D(x) \mid x\,\langle\!|D(x)|\!\rangle \\ = \; & x\,(y).(\ulcorner y \urcorner \mid x\,\langle\!|\ulcorner y \urcorner|\!\rangle) \mid x\,\langle\!|D(x)|\!\rangle \end{aligned}$$

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$$!P \triangleq D(x) \mid x\,\langle\!| P \mid D(x) |\!\rangle$$

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$$!P \triangleq D(x) \mid x\,\langle\!|P \mid D(x)|\!\rangle \rightarrow^* P \mid P \mid \ldots \mid !P$$

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$$D(x) \triangleq x\,(y)\,.\,(\ulcorner y \urcorner \mid x\,\langle\!|\ulcorner y \urcorner|\!\rangle)$$

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= \; & x\,(y)\,.\,(\ulcorner y \urcorner \mid x\,\langle\!|\ulcorner y \urcorner|\!\rangle) \mid x\,\langle\!|D(x)|\!\rangle \\
\to \; & D(x) \mid x\,\langle\!|D(x)|\!\rangle
\end{aligned}$$

$$!P \triangleq D(x) \mid x\,\langle\!|P \mid D(x)|\!\rangle$$
$$!u\,(v)\,.P \triangleq D(x) \mid x\,\langle\!|u\,(v)\,.\,(P \mid D(x))|\!\rangle$$

Example 2: Replication

$$D(x) \triangleq x(y).(\ulcorner y \urcorner \mid x\langle\!|\ulcorner y \urcorner|\!\rangle)$$

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$$!P \triangleq D(x) \mid x\langle\!|P \mid D(x)|\!\rangle$$
$$!u(v).P \triangleq D(x) \mid x\langle\!|u(v).(P \mid D(x))|\!\rangle \to$$
$$u(v).(P \mid D(x)) \mid x\langle\!|u(v).(P \mid D(x))|\!\rangle \not\to$$

Semantics

$$[\textsc{Par}]\,\frac{P_1 \to P_1'}{P_1 \mid P_2 \to P_1' \mid P_2}$$

$$[\textsc{Struct}]\,\frac{P_1 \equiv P_1' \quad P_1' \to P_2' \quad P_2' \equiv P_2}{P_1 \to P_2}$$

$$[\textsc{Com}]\,\frac{x_1 \equiv_{\mathcal{N}} x_2}{x_1 \langle\!| P_1 |\!\rangle \mid x_2(y).P_2 \to P_2\{\ulcorner P_1 \urcorner / y\}}$$

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$$[\textsc{N-drop}]\,\frac{x_1 \equiv_{\mathcal{N}} x_2}{\ulcorner\urcorner x_1 \ulcorner\urcorner \equiv_{\mathcal{N}} x_2}$$

$$[\textsc{N-struct}]\,\frac{P_1 \equiv P_2}{\ulcorner P_1 \urcorner \equiv_{\mathcal{N}} \ulcorner P_2 \urcorner}$$

Encoding the π-calculus

- We can encode output: $x{<}y{>} \triangleq x \langle\!| \ulcorner y \urcorner |\!\rangle$
- We can statically compose names:
  - $+x \triangleq \ulcorner x \langle\!| \mathbf{0} |\!\rangle \urcorner$
  - $x+ \triangleq \ulcorner x (\ulcorner \mathbf{0} \urcorner) . \mathbf{0} \urcorner$
  - $x \cdot y \triangleq \ulcorner x \langle\!| \mathbf{0} |\!\rangle \mid y (\ulcorner \mathbf{0} \urcorner) . \mathbf{0} \urcorner$

The encoding by Meredith & Radestock

$$[\![\mathbf{0}]\!]_{n,p} = \mathbf{0}$$

The encoding by Meredith & Radestock

$$\llbracket \mathbf{0} \rrbracket_{n,p} = \mathbf{0}$$

$$\llbracket \overline{x} \langle y \rangle \rrbracket_{n,p} = x \langle y \rangle$$

The encoding by *Meredith* & Radestock

$$[\![\mathbf{0}]\!]_{n,p} = \mathbf{0}$$

$$[\![\overline{x}{<}y{>}]\!]_{n,p} = x{<}y{>}$$

$$[\![x(y).P]\!]_{n,p} = x(y).[\![P]\!]_{n,p}$$

The encoding by Meredith & Radestock

$$
\begin{aligned}
[\![\mathbf{0}]\!]_{n,p} &= \mathbf{0} \\
[\![\overline{x}{<}y{>}]\!]_{n,p} &= x{<}y{>} \\
[\![x\,(\,y\,)\,.P]\!]_{n,p} &= x\,(\,y\,)\,.[\![P]\!]_{n,p} \\
[\![P_1 \mid P_2]\!]_{n,p} &= [\![P_1]\!]_{\color{red}{+n,+p}} \mid [\![P_2]\!]_{\color{red}{n+,p+}}
\end{aligned}
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[\![P_1 \mid P_2]\!]_{n,p} &= [\![P_1]\!]_{+n,+p} \mid [\![P_2]\!]_{n+,p+} \\
[\![(\nu x)P]\!]_{n,p} &= p\,(\,x\,)\,.[\![P]\!]_{\color{red}{+n,+p}} \mid p{<}n{>}
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[\![(\nu x)P]\!]_{n,p} &= p(x).[\![P]\!]_{+n,+p} \mid p{<}n{>} \\
[\![!P]\!]_{n,p} &= n \cdot p \left\langle\!\left| n{+}(n).p{+}(p).\left([\![P]\!]_{n,p} \mid D(n \cdot p) \mid n{+}\langle\!| n{<}n{>} |\!\rangle \mid p{+}\langle\!| p{<}p{>} |\!\rangle\right) \right|\!\right\rangle \\
&\quad \mid D(n \cdot p) \mid n{+}{<}{+}n{>} \mid p{+}{<}{+}p{>}
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[\![\mathbf{0}]\!]_{n,p} &= \mathbf{0} \\
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[\![P_1 \mid P_2]\!]_{n,p} &= [\![P_1]\!]_{+n,+p} \mid [\![P_2]\!]_{n+,p+} \\
[\![(\nu x)P]\!]_{n,p} &= p(x).[\![P]\!]_{+n,+p} \mid p\langle n\rangle \\
[\![\,!P]\!]_{n,p} &= n\cdot p\,\langle\!|\, n{+}(n).p{+}(p).\big([\![P]\!]_{n,p} \mid D(n\cdot p) \mid n{+}\langle\!|n\langle n\rangle|\!\rangle \mid p{+}\langle\!|p\langle p\rangle|\!\rangle\big)|\!\rangle \\
&\quad \mid D(n\cdot p) \mid n{+}\langle{+}n\rangle \mid p{+}\langle{+}p\rangle
\end{aligned}
$$

Claim (Meredith & Radestock):

$$P_1 \approx P_2 \iff [\![P_1]\!] \approx^{\mathrm{fn}(P_1)\cup\mathrm{fn}(P_2)} [\![P_2]\!]$$

where $P \downarrow^{\mathcal{N}} x_1 \triangleq x_1 \equiv_{\mathcal{N}} x_2 \wedge x_2 \in \mathcal{N} \wedge P \downarrow x_2$

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where $P \downarrow^{\mathcal{N}} x_1 \triangleq x_1 \equiv_{\mathcal{N}} x_2 \wedge x_2 \in \mathcal{N} \wedge P \downarrow x_2$

Unfortunately, that is not correct...

Intuitions

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$$\llbracket\, !P \rrbracket_{n,p} = n \cdot p \left\langle\!\!\left| n{+}\,(\,n\,)\,.p{+}\,(\,p\,)\,.\left(\llbracket P \rrbracket_{n,p} \;\mid\; D(n \cdot p) \;\mid\; n{+}\,\langle\!| n{<}n{>} |\!\rangle \;\mid\; p{+}\,\langle\!| p{<}p{>} |\!\rangle\right)\right|\!\!\right\rangle$$
$$\mid\; D(n \cdot p) \;\mid\; n{+}{<}{+}n{>} \;\mid\; p{+}{<}{+}p{>}$$

Intuitions

$$
\begin{aligned}
[\![\,!P]\!]_{n,p} &= n \cdot p \left\langle\!\left| n{+}\,(n)\,.p{+}\,(p)\,.\left([\![P]\!]_{n,p} \;\middle|\; D(n \cdot p) \;\middle|\; n{+}\,\langle\!| n{<}n{>} |\!\rangle \;\middle|\; p{+}\,\langle\!| p{<}p{>} |\!\rangle \right) \right|\!\right\rangle \\
&\quad \mid D(n \cdot p) \mid n{+}{<}{+}n{>} \mid p{+}{<}{+}p{>} \\
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$$
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&\quad \mid D(n \cdot p) \mid n{+}{<}{+}n{>} \mid p{+}{<}{+}p{>} \\
[\![(\nu x)P]\!]_{n,p} &= p\,(x)\,.[\![P]\!]_{+n,+p} \mid p{<}n{>} \\
[\![P_1 \mid P_2]\!]_{n,p} &= [\![P_1]\!]_{\color{red}{+n,+p}} \mid [\![P_2]\!]_{\color{red}{n+,p+}}
\end{aligned}
$$

A correct encoding

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$$!N(x, z, v, s) \triangleq D(x) \;\mid\; x \left\langle\!\left| z\,(a)\,.v\,(r)\,.\Big(D(x) \;\mid\; r\,\langle\!| \ulcorner a \urcorner |\!\rangle \;\mid\; z\,\langle\!| a\,\langle\!| \mathbf{0} |\!\rangle |\!\rangle\Big) \right|\!\right\rangle \;\mid\; z\,\langle\!| \ulcorner s \urcorner |\!\rangle$$

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$$!N(x,z,v,s) \triangleq D(x) \mid x\left\langle\!\left| z(a).v(r).\left(D(x) \mid r\langle\!|\ulcorner a \urcorner|\!\rangle \mid z\langle\!| a\langle\!|\mathbf{0}|\!\rangle |\!\rangle\right) \right|\!\right\rangle \mid z\langle\!|\ulcorner s \urcorner|\!\rangle$$

$$\begin{aligned}
[\![\mathbf{0}]\!]_{n,v} &= \mathbf{0} \\
[\![P_1 \mid P_2]\!]_{n,v} &= [\![P_1]\!]_{{\color{red}+n,v}} \mid [\![P_2]\!]_{{\color{red}n+,v}} \\
[\![x(y).P]\!]_{n,v} &= x(y).[\![P]\!]_{n,v} \\
[\![\overline{x}<z>]\!]_{n,v} &= x<z> \\
[\![(\nu x)P]\!]_{n,v} &= v<n> \mid n(x).[\![P]\!]_{n \cdot n,v} \\
[\![\,!x(y).P]\!]_{n,v} &= D(n) \mid n\left\langle\!\left| x(y).\left(D(n) \mid [\![P]\!]_{n \cdot n,v}\right) \right|\!\right\rangle
\end{aligned}$$

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[\![\mathbf{0}]\!]_{n,v} &= \mathbf{0} \\
[\![P_1 \mid P_2]\!]_{n,v} &= [\![P_1]\!]_{+n,v} \mid [\![P_2]\!]_{n+,v} \\
[\![x(y).P]\!]_{n,v} &= x(y).[\![P]\!]_{n,v} \\
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[\![(\nu x)P]\!]_{n,v} &= v\langle n\rangle \mid n(x).[\![P]\!]_{n\cdot n,v} \\
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[\![(\nu x)P]\!]_{n,v} &= v{<}n{>} \mid n(x).[\![P]\!]_{n \cdot n,v} \\
[\![\,!x(y).P]\!]_{n,v} &= D(n) \mid n \langle\!| x(y).\big(D(n) \mid [\![P]\!]_{n \cdot n,v}\big) |\!\rangle
\end{aligned}$$

Correctness criteria

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- Compositionality: $[\![S_1 \mid \ldots \mid S_n]\!]_N = C \mid [\![S_1]\!]_{N_1} \mid \ldots \mid [\![S_n]\!]_{N_n}$ where C is some context and $\mathrm{fn}\,(C) \subseteq \varphi(\mathrm{fn}\,(S_1 \mid \ldots \mid S_n)) \cup N$, and for each $i \in \{ 1, \ldots, n \}$ we have $N \rightsquigarrow N_i$.

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- Substitution invariance: $[\![S\sigma_s]\!]_N \simeq [\![S]\!]_N \sigma_t$ for each σ_s, where $\varphi(\sigma_s(x)) = \sigma_t(\varphi(x))$.

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- Substitution invariance: $[\![S\sigma_s]\!]_N \simeq [\![S]\!]_N\sigma_t$ for each σ_s, where $\varphi(\sigma_s(x)) = \sigma_t(\varphi(x))$.
- Operational correspondence: $S \rightarrow^* S' \iff \exists T' \,.\, [\![S]\!]_N \rightarrow^* T' \wedge T' \simeq [\![S']\!]_{N'}$ and $N \rightsquigarrow N'$.

Correctness criteria (cont.)

- Observational correspondence: We require that $N \cap \varphi(\mathscr{M}) = \emptyset$ for any set of observable names $\mathscr{M}$. Then $P \downarrow^{\mathscr{M}} \hat{x} \iff [\![P]\!]_N \Downarrow^{\varphi(\mathscr{M})} \varphi(\hat{x})$.

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- Divergence reflection: $[\![P]\!]_N \rightarrow^{\omega} \implies P \rightarrow^{\omega}$.
- Parameter independence: $[\![P]\!]_{N_1} \simeq [\![P]\!]_{N_2}$ for each finite N_1, N_2.

A separation result

The π-calculus cannot encode the ρ-calculus

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Thus: $P \not\downarrow u$ and $u \notin \mathrm{fn}(P)$

However: $P \to \ulcorner\urcorner x_1 \ulcorner \mid \urcorner x_2 \ulcorner\urcorner \langle\!| \mathbf{0} |\!\rangle = u \langle\!| \mathbf{0} |\!\rangle$ and $u \langle\!| \mathbf{0} |\!\rangle \downarrow u$.

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But the π-calculus cannot create new *free* names...

Conclusions

So what have we learned?

- Higher-order behaviour = reflection without modification
- The ρ-calculus *can* encode the π-calculus (modulo the criteria...)
- The π-calculus *cannot* encode the ρ-calculus (modulo the same criteria...)
- Names with *structure* requires some care
  ... especially with *parametrised* encodings!